Standard Practice for
Calculating Viscosity of a Blend of Petroleum Products[1]

ε¹ NOTE—Adjunct reference was added editorially in October 2007.

1. Scope

1.1 This practice covers the procedures for calculating the estimated kinematic viscosity of a blend of two or more petroleum products, such as lubricating oil base stocks, fuel components, residua with kerosine, crude oils, and related products, from their kinematic viscosities and blend fractions.

1.2 This practice allows for the estimation of the fraction of each of two petroleum products needed to prepare a blend meeting a specific viscosity.

1.3 This practice may not be applicable to other types of products, or to materials which exhibit strong non-Newtonian properties, such as viscosity index improvers, additive packages, and products containing particulates.

1.4 The values stated in SI units are to be regarded as standard. No other units of measurement are included in this standard.

1.5 Logarithms may be either common logarithms or natural logarithms, as long as the same are used consistently. This practice uses common logarithms. If natural logarithms are used, the inverse function, $\exp(\times)$, must be used in place of the base 10 exponential function, $10^{\times}$, used herein.

1.6 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and to determine the applicability of regulatory limitations prior to use.*

2. Referenced Documents

2.1 *ASTM Standards:*[2]

D341 Practice for Viscosity-Temperature Charts for Liquid Petroleum Products

D445 Test Method for Kinematic Viscosity of Transparent and Opaque Liquids (and Calculation of Dynamic Viscosity)

D7042 Test Method for Dynamic Viscosity and Density of Liquids by Stabinger Viscometer (and the Calculation of Kinematic Viscosity)

2.2 *ASTM Adjuncts:*

Calculating the Viscosity of a Blend of Petroleum Products Excel Worksheet[3]

3. Terminology

3.1 *Definitions of Terms Specific to This Standard:*

3.1.1 *ASTM Blending Method, n*—a blending method at constant temperature, using components in volume percent.

3.1.2 *blend fraction, n*—the ratio of the amount of a component to the total amount of the blend. Blend fraction may be expressed as mass percent or volume percent.

3.1.3 *blending method, n*—an equation for calculating the viscosity of a blend of components from the known viscosities of the components.

3.1.4 *dumbbell blend, n*—a blend made from components of widely differing viscosity.

3.1.4.1 *Example*—a blend of S100N and Bright Stock.

3.1.5 *inverse blending method, n*—an equation for calculating the predicted blending fractions of components to achieve a blend of given viscosity.

3.1.6 *mass blend fraction, n*—The ratio of the mass of a component to the total mass of the blend.

3.1.7 *McCoull-Walther-Wright Function, n*—a mathematical transformation of viscosity, generally equal to the logarithm of the logarithm of kinematic viscosity plus a constant, $\log[\log(v+0.7)]$. For viscosities below 2 mm²/s, additional terms are added to improve accuracy.

3.1.8 *modified ASTM Blending Method, n*—a blending method at constant temperature, using components in mass percent.

3.1.9 *modified Wright Blending Method, n*—a blending method at constant viscosity, using components in mass percent.

3.1.10 *volume blend fraction, n*—The ratio of the volume of a component to the total volume of the blend.

3.1.11 *Wright Blending Method, n*—a blending method at constant viscosity, using components in volume percent.

3.2 *Symbols:*

f_{ij} = blending fraction of component *i* calculated at temperature t_j. Blending fraction may be in mass percent or volume percent.

m_i = slope of the viscosity-temperature line, $\dfrac{(W_{i1} - W_{i0})}{(T_{i1} - T_{i0})}$

m_i^{-1} = reciprocal of the viscosity-temperature slope, m_i

t_B = temperature, in Celsius, at which the blend has viscosity v_B

t_{ij} = temperature, in Celsius, at which component i has viscosity v_{ij}

T_{ij} = transformed temperature
$$T_{ij} = log(273.15 + t_{ij}) \quad (1)$$

v_B = predicted kinematic viscosity of the blend, in mm^2/s, at temperature t_B if component blend fractions are known, or desired viscosity of the blend if component blend fractions are being calculated

v_{ij} = viscosity of component *i* at temperature t_j

W_{ij} = MacCoull-Walther-Wright function, a transformation of viscosity:
$$W_{ij} = log[log(v_{ij} + 0.7 + exp(-1.47 - 1.84 v_{ij} - 0.51 v_{ij}^2))] \quad (2)$$
where log is the common logarithm (base 10) and exp(x) is e (2.716...) raised to the power x.

W_H = arbitrary high reference viscosity, transformed using Eq 2

W_L = arbitrary low reference viscosity, transformed using Eq 2

4. Summary of Practice

4.1 The Wright Blending Method calculates the viscosity of a blend of components at a given temperature from the known viscosities, temperatures, and blending fractions of the components. The viscosities and temperatures of the components and the blend are mathematically transformed into MacCoull-Walther-Wright functions. The temperatures at which each component has two reference viscosities are calculated. The transformed reference temperatures are summed over all components as a weighted average, with the blend fractions as the weighting factors. The two temperatures at which the blend has the reference viscosities are used to calculate the blend viscosity at any other temperature.

4.2 The Inverse Wright Blending Method calculates the blend fractions of components required to meet a target blend viscosity from the known viscosities and temperatures of the components. The viscosities and temperatures of the components and the blend are mathematically transformed into MacCoull-Walther-Wright functions. The temperatures at which each component has the target blend viscosity are calculated. The component transformed temperatures are summed over all components, as a weighted average, to meet the target blend transformed temperature. The weighting factors are the desired blend fractions, which are obtained by inverting the weighted summation equation.

4.3 The ASTM Blending Method calculates the viscosity of a blend of components at a given temperature from the known viscosities of the components at the same temperature and their blending fractions. The viscosities of the components and the blend are mathematically transformed into MacCoull-Walther-Wright functions. The transformed viscosities are summed over all components as a weighted average, with the blend fractions as the weighting factors. The transformed viscosity is untranformed into viscosity units.

4.4 The Inverse ASTM Blending Method calculates the blend fractions of components required to meet a target blend viscosity at a given temperature from the known viscosities of the components at the same temperature. The viscosities of the components and the blend are mathematically transformed into MacCoull-Walther-Wright functions. The component transformed viscosities are summed over all components, as a weighted average, to equal the target blend transformed viscosity. The weighting factors are the desired blend fractions, which are obtained by inverting the weighted summation equation.

5. Significance and Use

5.1 Predicting the viscosity of a blend of components is a common problem. Both the Wright Blending Method and the ASTM Blending Method, described in this practice, may be used to solve this problem.

5.2 The inverse problem, predicating the required blend fractions of components to meet a specified viscosity at a given temperature may also be solved using either the Inverse Wright Blending Method or the Inverse ASTM Blending Method.

5.3 The Wright Blending Methods are generally preferred since they have a firmer basis in theory, and are more accurate. The Wright Blending Methods require component viscosities to be known at two temperatures. The ASTM Blending Methods are mathematically simpler and may be used when viscosities are known at a single temperature.

5.4 Although this practice was developed using kinematic viscosity and volume fraction of each component, the dynamic viscosity or mass fraction, or both, may be used instead with minimal error if the densities of the components do not differ greatly. For fuel blends, it was found that viscosity blending using mass fractions gave more accurate results. For base stock blends, there was no significant difference between mass fraction and volume fraction calculations.

5.5 The calculations described in this practice have been computerized as a spreadsheet and are available as an adjunct.[3]

6. Procedure

Procedure A

6.1 *Calculating the Viscosity of a Blend of Components With Known Blending Fractions by the Wright Blending Method:*

6.1.1 This section describes the general procedure to predict the viscosity of a blend, given the viscosity-temperature properties of the components and their blend fractions. Any number of components may be included. If the blend fractions are in volume percent, this is known as the Wright Blending Method. If the blend fractions are in mass percent, this is known as the Modified Wright Blending Method.

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6.1.2 Compile, for each component, its blend fraction, and viscosities at two temperatures. The viscosity of component i at temperature t_{ij} is designated v_{ij}, and its blend fraction is f_i. If the viscosities are not known, measure them using a suitable test method. The two temperatures may be the same or different for each component.

NOTE 1—Test Methods D445 and D7042 have been found suitable for this purpose.

6.1.3 Transform the viscosities and temperatures of the components as follows:

$$Z_{ij} = v_{ij} + 0.7 + \exp(-1.47 - 1.84v_{ij} - 0.51v_{ij}^2) \quad (3)$$

$$W_{ij} = \log[\log(Z_{ij})] \quad (4)$$

$$T_{ij} = \log[t_{ij} + 273.15] \quad (5)$$

where v_{ij} is the kinematic viscosity, in mm²/s, of component i at temperature t_{ij} in degrees Celsius, exp() is e (2.716) raised to the power x, and log is the common logarithm (base 10).

6.1.3.1 If the kinematic viscosity is greater than 2 mm²/s, the exponential term in Eq 3 is insignificant and may be omitted.

6.1.3.2 Transform the temperature at which the blend viscosity is desired using Eq 5. This transformed temperature is designated T_B.

6.1.4 Calculate the inverse slope for each component, as follows:

$$m_i^{-1} = \frac{(T_{i1} - T_{i0})}{(W_{i1} - W_{i0})} \quad (6)$$

6.1.5 Calculate the predicted transformed viscosity, W_B, of the blend at temperature T_B, as follows:

$$W_B = \frac{T_B + \Sigma f_i (m_i^{-1} W_{i0} - T_{i0})}{\Sigma (f_i m_i^{-1})} \quad (7)$$

where the sum is over all components.

6.1.6 Calculate the untransformed viscosity of the blend, v_B, at the given temperature:

$$Z'_B = 10^{W_B} \quad (8)$$

$$Z_B = 10^{Z'_B} - 0.7 \quad (9)$$

$$v_B = Z_B - \exp[-0.7487 - 3.295Z_B + 0.6119Z_B^2 - 0.3193Z_B^3] \quad (10)$$

where Z'_B and Z_B are the results of intermediate calculation steps with no physical meaning.

NOTE 2—For viscosities between 0.12 and 1000 mm²/s, the transforming Eq 3 and Eq 4 and the untransforming equations Eq 9 and Eq 10 have a discrepancy less than 0.0004 mm²/s.

NOTE 3—See the worked example in Appendix X3.

Procedure B

6.2 *Calculating the Blend Fractions of Components to Give a Target Viscosity Using the Inverse Wright Blending Method*:

6.2.1 This section describes the general procedure to predict the required blending fractions of two components to meet a target blend viscosity at a given temperature, given the

viscosity-temperature properties of the components. This is known as the Inverse Wright Blending Method.

6.2.1.1 In principle, the blend fractions may be calculated for more than two blending components, if additional constraints are specified for the final blend. Such calculations are beyond the scope of this practice.

6.2.2 Compile the viscosities of the components at two temperatures each. The viscosity of component i at temperature t_{ij} is designated v_{ij}. If the viscosities are not known, measure them using a suitable test method. The two temperatures do not have to be the same for both components, nor do they have to be the same as the temperature at which the target viscosity is specified.

NOTE 4—Test Methods D445 and D7042 have been found suitable for this purpose.

6.2.3 Transform the viscosities and temperatures of the components using Eq 3, Eq 4, and Eq 5.

6.2.4 Use the target blend viscosity, v_B, as a reference viscosity. Transform v_B to W_B using equations Eq 3 and Eq 4.

6.2.5 Calculate the transformed temperatures at which each component has that viscosity:

$$T_{iL} = \frac{(T_{i1} - T_{i0})}{(W_{i1} - W_{i0})}(W_L - W_{i0}) + T_{i0} \quad (11)$$

6.2.6 Calculate the predicted blend fraction of the first component:

$$f_1 = \frac{(T_B - T_{0L})}{(T_{1A} - T_{0L})} \quad (12)$$

and the fraction of the second component is $f_2 = (1 - f_1)$ because the total of the two components is 100 %.

NOTE 5—See the worked example in Appendix X4.

Procedure C

6.3 *Calculating the Viscosity of a Blend of Components With Known Blending Fractions Using the ASTM Blending Method*:

6.3.1 This section describes the general procedure to predict the viscosity of a blend at a given temperature, given the viscosities of the components at the same temperature and their blend fractions. Any number of components may be included. If the blend fractions are in volume percent, this is known as the ASTM Blending Method. If the blend fractions are in mass percent, this is known as the Modified ASTM Blending Method.

6.3.2 Compile the viscosities of the components at a single temperature (the reference temperature). The viscosity of component i at that temperature is designated v_i. If the viscosities are not known, measure them using a suitable test method.

NOTE 6—Test Methods D445 and D7042 have been found suitable for this purpose.

6.3.2.1 If the viscosity of a component is not known at the reference temperature, but is known at two other temperatures, use Viscosity-Temperature Charts D341 or Eq 10 to calculate its viscosity at the reference temperature.

6.3.3 Transform the viscosities of the components using Eq 2.

6.3.4 Calculate the transformed viscosity of the blend as a weighted average of the component transformed viscosities, using the blend fractions as the weighting factors:

$$W_B = \frac{\sum [f_i W_i]}{\sum [f_i]} \tag{13}$$

where W_B is the transformed viscosity of the blend, f_i is the blend fraction of component i, and W_i is the transformed viscosity of component $_i$.

6.3.4.1 Normally, the sum of blend fractions is 100 %:

$$\sum (f_i) = 1 \tag{14}$$

and the denominator in Eq 12 may be omitted. However, the more general formula may be used when more convenient, for example to save re-normalizing the base stock fractions in an oil containing other components (for example, additives).

6.3.5 Calculate the predicted (untransformed) viscosity of the blend at the reference temperature:

$$v_B = (W_B - 0.7) - \exp[-0.7487 - 3.295(W_B - 0.7) + 0.6119(W_B - 0.7)^2 - 0.3193(W_B - 0.7)^3] \tag{15}$$

Note 7—See the worked example in Appendix X5.

Procedure D

6.4 —Calculating the Blend Fractions of Components to Give a Target Viscosity using the Inverse ASTM Blending Method

6.4.1 This section describes the general procedure to predict the required blending fractions of two components to meet a target blend viscosity at a given temperature, given the viscosity of the components at the same temperature. This is known as the Inverse ASTM Blending Method.

6.4.1.1 In principle, the blend fractions may be calculated for more than two blending components, if additional constraints are specified for the final blend. Such calculations are beyond the scope of this practice.

6.4.2 Compile the viscosities of the components at the temperature at which the target blend viscosity is specified. The viscosity of component i at this temperature is designated v_i. If the viscosities are not known, measure them using a suitable test method. If the viscosity of a component is not known at the reference temperature, but is known at two other temperatures, calculate the viscosity at the reference temperature using Viscosity-Temperature Charts D341 or Eq 10.

Note 8—Test Methods D445 and D7042 have been found suitable for this purpose.

6.4.3 Transform the viscosities of the components and the target blend using Eq 4.

6.4.4 Calculate the blend fraction of the first component:

$$f_1 = \frac{(W_B - W_2)}{(W_1 - W_2)} \tag{16}$$

where W_i is the transformed viscosity of component i at the given temperature and f_1 is the blending fraction of component 1. The blending fraction of the second component is $f_2 = (1 - f_1)$ because the total of the two components is 100 %.

Note 9—See the worked example in Appendix X6.

7. Report

7.1 Report the predicted viscosity of the blend at the given temperature, if known blending fractions were given.

7.2 Report the calculated blending fractions, if a target blend viscosity was given.

7.3 Report which procedure was used for the calculation.

8. Measurement Uncertainty

8.1 The calculations in this practice are exact, given the input data.

8.2 Measuring or compiling the input data can introduce sources of variation. For example, the measured viscosities of the components will vary according the precision of Test Methods D445 or D7042, and will lead to variation in calculated results using this practice. For the Wright Blending Methods, measuring viscosities at narrowly-spaced temperatures is expected to lead to greater variability than using widely spaced temperatures, due to the increased uncertainty in the slope of the fitted viscosity-temperature equations.

8.3 Agreement between the methods in this practice and experimental results were determined in two studies.

8.3.1 The agreements between calculated and predicted results were compared for 37 fuel blends.[4] The fuel components included light gas oil, heavy gas oil, light cat cracked cycle oil, bright stock furfural extract, kerosine, and short residue. For fuel blends, the agreement is close to the precision of the experimental data and viscosity blending using mass fractions is preferred to volume fractions. Using mass fractions, all blend combinations show a positive bias (calculated higher than actual). The bias is dependent on blending ratio and clearly indicates that viscosity blending is non-linear. The magnitude of the bias seems to be correlated with the absolute density difference and the viscosity difference between the blend components.

8.3.1.1 For fuels, the difference between calculated and measured blend viscosities is expected to exceed the following values only one time in twenty:

Blending Basis	N	ASTM Method		Wright Method	
		Mass%	Vol%	Mass%	Vol%
Fuels	21	30%	47%	14	21%
Fuels – Near Blends	12	5%	4%	4%	8%

with the greatest disagreement around equal proportions of components (for example, 50:50). "Near blends" refers to binary blends of adjacent viscosity streams (for example, light gas oil and heavy gas oil or light cycle cat cracker oil and heavy gas oil), and excludes "dumbbell" blends (for example, light cycle cat cracker oil and bright stock furfural extract, or kerosine and short residuum).

8.3.2 The agreements between calculated and predicted results were compared for 30 base stock blends.[5] The base stock components included S100N, S150N, S600N, and Bright Stock. For base stock blends, the difference between calculated and experimental results are expected to exceed the following values only one time in twenty:

Blending Basis		ASTM Method		Wright Method	
	N	Mass%	Vol%	Mass%	Vol%
Base Stocks	30	10%	12%	1%	2%
Base Stocks – Near Blends	14	4%	4%	1%	1%

with the greatest disagreement around equal proportions of components (for example, 50:50; 33:33:33; or 25:25:25:25). "Near blends" refers to binary blends of adjacent viscosity grades (for example, S100N and S150N), and excludes "dumb-bell" blends (for example, S100N and Bright Stock).

9. Keywords

9.1 blending; kinematic viscosity; MacCoull; viscosity; Wright

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[5] Supporting data have been filed at ASTM International Headquarters and may be obtained by requesting Research Report RR:D02-1574.

APPENDIXES

(Nonmandatory Information)

X1. RATIONALE

X1.1 A method to calculate the viscosity-temperature properties of a blend, given the viscosity-temperature properties of the components and their blending fraction was first published by Wright.[6] A graphical procedure was included as an Appendix (nonmandatory information) in Viscosity-Temperature Charts D341 for many years. For companies using these methods, this led to conflicts with auditors who questioned compliance with a nonmandatory procedure. This could not be resolved by moving the calculations to an Annex (Mandatory Information) because not all users of Viscosity-Temperature Charts D341 intended to perform blending calculations. A separate practice was needed.

X1.2 The graphical methods of Viscosity-Temperature Charts D341 have been superseded by computational methods using calculators or spreadsheets. The graphical method is not included in this practice and an electronic adjunct was developed.[3]

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[6] W. Andrew Wright, "Prediction of Oil Viscosity Blending," American Chemical Society, Atlantic City Meeting, April 8-12, 1946.

X2. DERIVATION OF WRIGHT BLENDING METHOD EQUATIONS

X2.1 This appendix gives the derivation of the equations used to compute blend viscosities according to the Wright Blending Method.

X2.2 The concept of the Wright Blending Method is illustrated in Fig. X2.1. For simplicity, a blend comprising only two components is shown, although the equations will be derived for the general case of n components. The x-axis is transformed temperature and the y-axis is transformed viscosity. The transformations for temperature and viscosity are given by, respectively, Eq 1 and 2 of this practice. Line 1 shows the viscosity-temperature relationship of component 1, determined from the two known viscosity-temperature data points, (T_{10}, W_{10}) and (T_{11}, W_{11}). Line 2 shows the same data for component 2. The blend fractions of components 1 and 2 are denoted by, respectively, f_1 and f_2; these may be in either mass percent or volume percent. The Wright Blending Method will determine the viscosity-temperature relationship of the blend, Line B, from these data.

X2.3 Each line has the equation:

$$W_{ij} = m_i X_{ij} + b_i \qquad \text{(X2.1)}$$

which is the slope-intercept equation of a straight line, where the subscript i indicates the oil (components or blend), and the subscript j indicates the particular temperature-viscosity point.

The parameters m and b are the slope and intercept, respectively.

X2.4 These straight lines may also be expressed using the point-point equation:
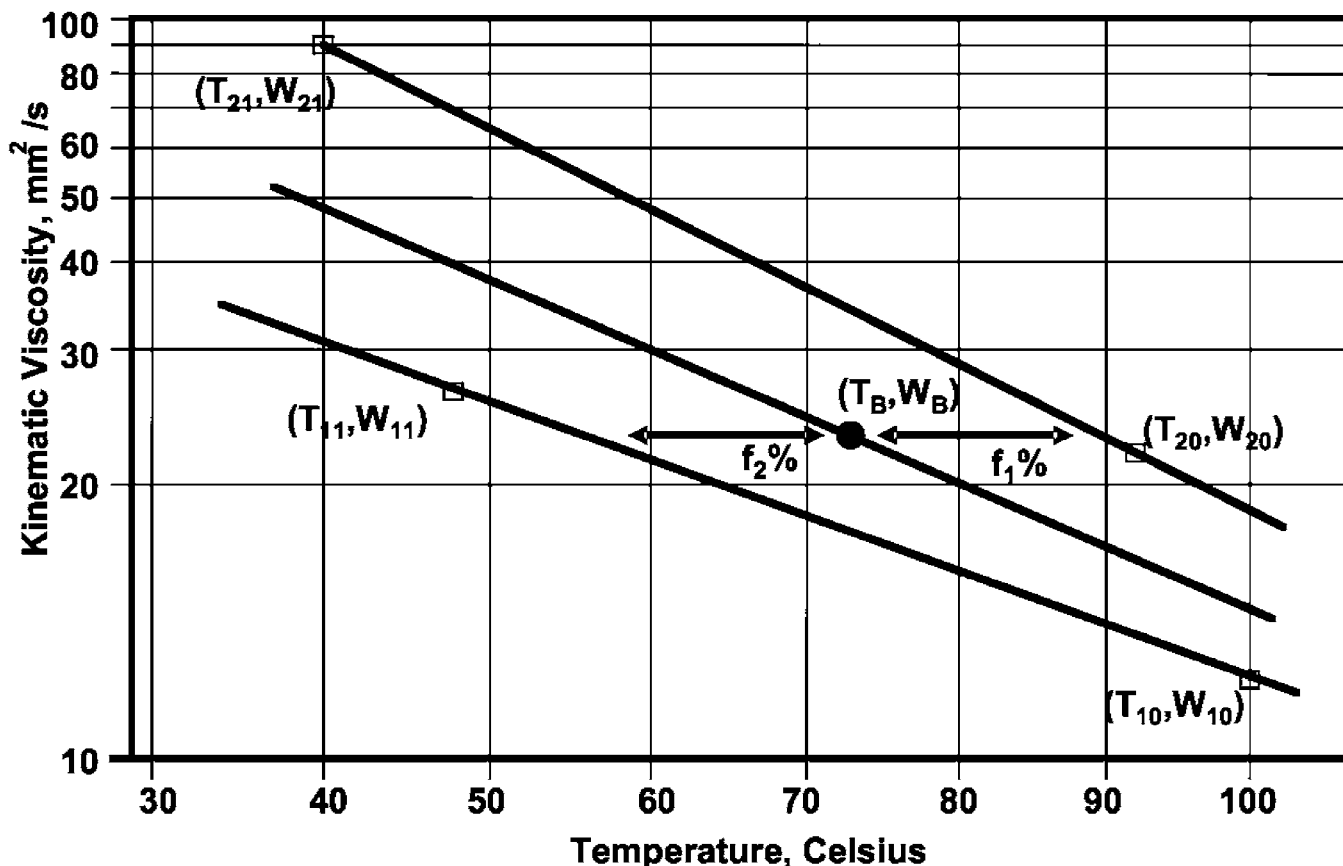
$$W_{ij} = \left(\frac{W_{i1} - W_{i0}}{T_{i1} - T_{i0}} \right)(T_{ij} - T_{i0}) + W_{i0} \qquad \text{(X2.2)}$$

where (T_{i0}, W_{i0}) and (T_{i1}, W_{i1}) are two known temperature-viscosity points for component i, and subscript j indicates the specific point to be determined (for example, a particular temperature or viscosity).

X2.5 The first expression in brackets represents the slope of the line:

$$m_i = \left(\frac{W_{i1} - W_{i0}}{T_{i1} - T_{i0}} \right) \qquad \text{(X2.3)}$$

and its reciprocal is designated m_i^{-1}:

Points (T_{10}, W_{10}), and (T_{11}, W_{11}) are the two known temperature-viscosity points for blend component 1. f_1 is the blend fraction of component 1.
Points (T_{20}, W_{20}) and (T_{21}, W_{21}) are the two known temperature-viscosity points for blend component 2. f_2 is the blend fraction of component 2.
In general, points (T_{i0}, W_{i0}) and (T_{i1}, W_{i1}) are the two known temperature-viscosity points for blend component i. f_i is the blend fraction of component i.
T_B is the (transformed) temperature at which the viscosity of the blend is sought.
W_B is the calculated (transformed) viscosity of the blend at temperature T_B.

FIG. X2.1 Schematic Illustration of Wright Blending Method

$$m_i^{-1} = \left(\frac{T_{i1} - T_{i0}}{W_{i1} - W_{i0}} \right) \qquad \text{(X2.4)}$$

X2.6   The temperature corresponding to a given viscosity for a given component is:

$$T_{ij} = m_i^{-1}(W_{ij} - W_{i0}) + T_{i0} \qquad \text{(X2.5)}$$

X2.7   It is convenient to define two reference (transformed) viscosities, W_L and W_U, representing lower and upper points on Line B. These reference viscosities are arbitrary and will later be factored out of the final equation.

X2.8   The temperature at which each component has transformed viscosity W_L is:

$$T_{iL} = m_i^{-1}(W_L - W_{i0}) + T_{i0} \qquad \text{(X2.6)}$$

and likewise for viscosity W_U.

X2.9   The Wright Blending Method combines temperatures at a given viscosity as a weighted average. The temperature at which the blend has viscosity W_L is:

$$T_{BL} = \frac{\sum_{i=1}^{n} (f_i T_{iL})}{\sum_{i=1}^{n} f_i} \qquad \text{(X2.7)}$$

where the sum is over all components. There is a similar equation for temperature T_U, at which the blend has viscosity W_U.

X2.10   Substituting equations Eq X2.6 into equation Eq X2.7 gives:

$$T_{BL} = \frac{W_L \sum_{i=1}^{n} (f_i m_i^{-1}) - \sum_{i=1}^{n} f_i(m_i^{-1}W_{i0} - T_{i0})}{\sum_{i=1}^{n} f_i} \qquad \text{(X2.8)}$$

and similarly for temperature T_U.

X2.11 Now, two temperature-viscosity points are known for the blend, (T_{BL}, W_L) and (T_{BU}, W_U). These are substituted into Eq X2.2 to determine the viscosity of the blend at the required temperature:

$$W_{Bj} = \left(\frac{W_U - W_L}{T_{BU} - T_{BL}}\right)(T_{Bj} - T_{BL}) + W_L \qquad (X2.9)$$

X2.12 A few pages of algebra gives:

$$W_{Bj} = \frac{T_B + \sum f_i \, (m_i^{-1} W_{i0} - T_{i0})}{\sum (f_i m_i^{-1})} \qquad (X2.10)$$

which is the fundamental equation for the Wright Blending Method.

X3. WORKED EXAMPLE OF THE WRIGHT BLENDING METHOD

X3.1 This appendix gives a worked example of calculating the expected viscosity of a blend of components, using the Wright Blending Method.

X3.2 Base stock A has kinematic viscosities at 80°C and 40°C of, respectively, 5 mm^2/s and 30 mm^2/s. Base stock B has kinematic viscosities at 100°C and 35°C of, respectively, 12 mm^2/s and 112 mm^2/s. What is the expected viscosity at 50°C of a 60:40 blend of A and B?

X3.3 Transform the viscosity-temperature data:

$$\begin{aligned} W_{10} &= \log[\log(5 + 0.7 + \exp(-1.47 - 1.84 \cdot 5 - 0.51 \cdot 5^2))] \\ &= \log[\log(5.70)] = -0.1216 \qquad (X3.1) \end{aligned}$$

$$\begin{aligned} W_{11} &= \log[\log(30 + 0.7 + \exp(-1.47 - 1.84 \cdot 30 - 0.51 \cdot 30^2))] \\ &= \log[\log(30.70)] = 0.1724 \qquad (X3.2) \end{aligned}$$

$$\begin{aligned} W_{20} &= \log[\log(12 + 0.7 + \exp(-1.47 - 1.84 \cdot 12 - 0.51 \cdot 12^2))] \\ &= \log[\log(12.70)] = 0.0429 \qquad (X3.3) \end{aligned}$$

$$\begin{aligned} W_{21} &= \log[\log(112 + 0.7 + \exp(-1.47 - 1.84 \cdot 112 - 0.51 \cdot 112^2))] \\ &= \log[\log(112.70)] = 0.3122 \qquad (X3.4) \end{aligned}$$

$$T_{10} = \log[80 + 273.15] = \log[353.15] = 2.5480$$

$$T_{11} = \log[40 + 273.15] = \log[313.15] = 2.4958 \qquad (X3.5)$$

$$T_{20} = \log[100 + 273.15] = \log[373.15] = 2.5719 \qquad (X3.6)$$

$$T_{21} = \log[35 + 273.15] = \log[308.15] = 2.4888 \qquad (X3.7)$$

$$T_B = \log[50 + 273.15] = \log[323.15] = 2.5094 \qquad (X3.8)$$

X3.4 Calculate the inverse slope for each component:

$$m_1^{-1} = \frac{(2.4958 - 2.5480)}{(0.1724 - (-0.1216))} = \frac{-0.0522}{0.2939} = -0.1776 \qquad (X3.9)$$

$$m_2^{-1} = \frac{(2.4888 - 2.5719)}{(0.3122 - (-0.0429))} = \frac{-0.0831}{0.2693} = -0.3087 \qquad (X3.10)$$

X3.5 Calculate the transformed viscosity of the blend at 50°C:

$$W_{Bj} = \frac{T_B + \sum f_i \, (m_i^{-1} W_{i0} - T_{i0})}{\sum (f_i m_i^{-1})} \qquad (X3.11)$$

$$= \frac{2.5094 + (0.6)[(-0.1776)(0.1724) - 2.4958] + (0.4)[(-0.3087)(0.3122) - 2.488]}{(0.6)(-0.1776) + (0.4)(-0.3087)}$$

$$= \frac{2.5094 + (0.6)[-2.5264] + (0.4)[-2.5851]}{(-0.1066) + (-0.1235)}$$

$$= \frac{2.5094 - 1.5158 - 1.0340}{-0.2301} = \frac{-0.0405}{-0.2301} = 0.1759$$

X3.6 Calculate the untransformed viscosity of the blend at 50°C:

$$Z'_B = 10^{W_B} = 10^{0.1759} = 1.499 \qquad (X3.12)$$

$$Z_B = 10^{Z'_B} - 0.7 = 10^{1.499} - 0.7 = (31.57 - 0.7) = 30.87 \qquad (X3.13)$$

$$\begin{aligned} v_B &= 30.87 - \exp[-0.7487 - 3.295(30.87) + 0.6119(30.87)^2 \\ &\quad - 0.3193(30.87)^3] \qquad (X3.14) \end{aligned}$$

$$\begin{aligned} &= 30.87 - \exp[-0.7487 - 101.7288 + 583.2533 - 9396.4553] \\ &= 30.87 - \exp[-8916] = (30.87 - 0) = 30.87 \end{aligned}$$

and the predicted viscosity of the blend at 50°C is 30.87 mm^2/s.

NOTE X3.1—The inverse problem is worked in Appendix X4.

X4. WORKED EXAMPLE OF THE INVERSE WRIGHT BLENDING METHOD

X4.1   This appendix gives a worked example of calculating the expected blend fractions for two components to meet a target blend viscosity, using the Inverse Wright Blending Method.

X4.2   Base stock A has kinematic viscosities at 80°C and 40°C of, respectively, 5 mm^2/s and 30 mm^2/s. Base stock B has kinematic viscosities at 100°C and 35°C of, respectively, 12 mm^2/s and 112 mm^2/s. What are the relative proportions of A and B to make a blend viscosity of 31 mm^2/s at 50°C?

X4.3   Calculate the temperature at which base stock A has a viscosity of 31 mm^2/s:

$$T_{A,31} = \left(\frac{(\log(273.15 + 80) - \log(273.15 + 40))}{(\log[\log(5.7)] - \log[\log(30.7)])}\right) \cdot (\log[\log(31.7)] - \log[\log(5.7)]) + \log(273.15 + 80) \quad \text{(X4.1)}$$

$$T_{A,31} = \left(\frac{(2.5480 - 2.4958)}{(-0.1216 - 0.1724)}\right) \cdot (0.1764 - (-0.1216)) + 2.5480 \quad \text{(X4.2)}$$

$$T_{A,31} = \left(\frac{(0.0522)}{(-0.2939)}\right) \cdot 02979 + 2.5480$$
$$= -0.0529 + 2.5094$$
$$= 2.4950 \quad \text{(X4.3)}$$

and $t_{A,31} = 39.48$°C, although it is not necessary to untransform this temperature.

X4.4   Calculate the temperature at which base stock B has a viscosity of 31 mm^2/s:

$$T_{B,31} = \left(\frac{\log(273.15 + 100) - \log(273.15 + 35))}{(\log[\log(12.7)] - \log[\log(112.7)])}\right) \cdot$$
$$(\log[\log(31.7)[- \log[\log(12.7)]) + \log(273.15 + 100) \quad \text{(X4.4)}$$

$$T_{A,31} = \left(\frac{(2.5719 - 2.4888)}{(0.0429 - 0.3122)}\right) \cdot (0.1764 - 0.0429) + 2.5719 \quad \text{(X4.5)}$$

$$T_{A,60} = \left(\frac{(0.0831)}{(-0.2693)}\right) \cdot 0.1335 + 2.5719$$
$$= -0.0412 + 2.5719$$
$$= 2.5307 \quad \text{(X4.6)}$$

and $t_{B,31} = 66.22$°C.

X4.5   Calculate the blending fraction of base stock A:

$$f_1 = \frac{(T_{Blend} - T_2)}{(T_1 - T_2)} \quad \text{(X4.7)}$$

$$f_1 = \frac{(\log(273.15 + 50) - \log(273.15 + 66.22))}{(\log(273.15 + 39.48) - \log(273.15 + 66.22))} \quad \text{(X4.8)}$$

$$f_1 = \frac{(2.5094 - 2.5307)}{(2.4950 - 2.5307)} = \frac{-0.0213}{-0.0356} = 0.60 \quad \text{(X4.9)}$$

and the blend fraction of base stock A is 60 %. The blend fraction of base stock B is $(1 - f_A) = 40$ %, and this calculation agrees, within rounding, with the result in Appendix X3.

X5. WORKED EXAMPLE OF ASTM BLENDING METHOD

X5.1   This appendix gives a worked example of calculating the expected viscosity of a blend, given the component properties and blending fractions, using the ASTM Blending Method.

X5.2   Base stock A has kinematic viscosity at 100°C of 6 mm^2/s. Base stock B has kinematic viscosity at 100°C of 8 mm^2/s. What is the viscosity at 100°C of a blend made from 25 % base stock A and 75 % base stock B?

X5.3   Transform the base stock viscosities, using Eq 3.

$$W_A = log[log(6 + 0.7)] = -0.0830 \quad \text{(X5.1)}$$

$$W_B = log[log(8 + 0.7)] = -0.0271 \quad \text{(X5.2)}$$

X5.4   Add the transformed viscosities, weighted by the blend fractions:

$$W_{Blend} = 0.25 \cdot W_A + 0.75 \cdot W_B$$
$$= 0.25(-0.0830) + 0.75(-0.0271)$$
$$= -0.0411 \quad \text{(X5.3)}$$

and

$$v_{Blend} = 10^{10^{Z_{Blend}}} - 0.7$$
$$= 10^{10^{-0.0411}} - 0.7$$
$$= 10^{0.9098} - 0.7$$
$$= 8.12 - 0.7$$
$$= 7.42 \quad \text{(X5.4)}$$

NOTE X5.1—The inverse problem is worked in Appendix X6.

X6. WORKED EXAMPLE OF THE INVERSE ASTM BLENDING METHOD

X6.1   This appendix gives a worked example of calculating the expected blend fractions for two components to meet a target blend viscosity, using the Inverse ASTM Blending Method.

X6.2   Base stock A has kinematic viscosity at 100°C of 6 mm^2/s. Base stock B has kinematic viscosity at 100°C of 8 mm^2/s. What are the blending fractions required to make a blend with a viscosity of 7.4 mm^2/s at 100°C?

X6.3   Transform the base stock viscosities, using Eq 3.

$$W_A = log[log(6 + 0.7)] = -0.0830 \qquad (X6.1)$$

$$W_B = log[log(8 + 0.7)] = -0.0271 \qquad (X6.2)$$

$$W_{Blend} = log[log(7.4 + 0.7) = -0.0417 \qquad (X6.3)$$

X6.4   Calculate the predicted blend fraction of component A:

$$f_A = \frac{(W_{Blend} - W_B)}{(W_A - W_B)} \qquad (X6.4)$$

$$f_A = \frac{(-0.0417) - (-0.0271)}{(-0.0830) - (-0.0271)} = \frac{-0.0146}{-0.0559} = 0.26 \qquad (X6.5)$$

and the blend fraction of component A is 26 %. By subtraction, the blend fraction of component B is 74 %. This agrees, within rounding, with the result in Appendix X5.