Standard Practice for
Calculation of Pore Size Distributions of Catalysts from Nitrogen Desorption Isotherms[1]

1. Scope

1.1 This practice covers the calculation of pore size distributions for catalysts and catalyst carriers from nitrogen desorption isotherms. The computational procedure is particularly useful for determining how the pore volume is distributed in catalyst samples containing pores whose sizes range from approximately 1.5 to 100 nm (15 to 1000 Å) in radius. It should be used with caution when applied to isotherms for samples containing pores both within this size range and pores larger than 100 nm (1000 Å) in radius. In such instances the isotherms rise steeply near $P/P_o = 1$ and the total pore volume cannot be well defined. The calculations should be begun at a point on the isotherm near saturation preferably in a region near $P/P_o = 0.99$, establishing an upper limit on the pore size distribution range to be studied. Simplifications are necessary regarding pore shape. A cylindrical pore model is assumed, and the method treats the pores as non-intersecting, open-ended capillaries which are assumed to function independently of each other during the adsorption or desorption of nitrogen.

NOTE 1—This practice is designed primarily for manual computation and a few simplifications have been made for this purpose. For computer computation, the simplified expressions may be replaced by exact expressions.

1.2 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*

2. Referenced Documents

2.1 *ASTM Standards:[2]*

D3766 Terminology Relating to Catalysts and Catalysis

D4222 Test Method for Determination of Nitrogen Adsorption and Desorption Isotherms of Catalysts and Catalyst Carriers by Static Volumetric Measurements

3. Terminology

3.1 *Definitions*—Consult Terminology D3766.

3.2 *Symbols:*

$P_4(i)$	=	pressure after equilibration during desorption, torr.
$P_0(i)$	=	liquid nitrogen vapor pressure, torr.
V_{de}	=	see 12.4.10 and 12.5 in Test Method D4222.
$r_k(i)$	=	radius of inner core calculated from Kelvin equation, Å.
T	=	boiling point of nitrogen, K.
V_L	=	liquid nitrogen molar volume at T, cm^3/mole.
γ	=	liquid nitrogen surface tension at T, mN/m.
$T(i)$	=	average thickness of the nitrogen film adsorbed on the pore walls, Å.
$r_p(i)$	=	radius of cylindrical pore given by $r_k(i) + t(i)$, Å.
Q	=	volume correction factor defined as $(\bar{r}_p/\bar{r}_k)^2$.
$\Delta V_T(i)$	=	decrease in the amount of nitrogen adsorbed caused by a lowering in relative pressure, mm^3/g.
$\Delta V_f(i)$	=	volume of liquid nitrogen desorbed from pore walls during thinning of the film, mm^3/g.
$\Delta V_k(i)$	=	liquid volume of the inner core in which capillary condensation of the nitrogen occurs, mm^3/g.
$\Delta V_p(i)$	=	liquid volume contained in a group of pores having mean radius $\bar{r}_p$, mm^3/g.
$\Sigma\Delta v_p$	=	cumulative pore volume, mm^3/g.
$\Delta S_p(i)$	=	area of the pore walls of a cylinder having volume ΔV_p, m^2/g.

4. Summary of Practice

4.1 The pore size distribution is determined by analyzing the desorption data of the nitrogen isotherm. The nitrogen uptake is caused by the multilayer adsorption of a film of nitrogen on the pore walls and by capillary condensation of the nitrogen in the "inner core" regions of the pores. The relative pressure at which filling of the core occurs for a given pore size

by capillary condensation is predicted from the Kelvin equation (**1**).[3] During desorption, thinning of the multilayer film adsorbed on the pore walls occurs in pores which have previously lost their capillary condensate. Corrections for film thinning are determined by a procedure involving the surface area and radius of the film which becomes exposed as desorption proceeds. In principle, the computational procedure can be applied to either the adsorption branch or desorption branch of the nitrogen isotherm. Unless the presence of ink-bottle shaped pores is suggested by an abrupt closure of the desorption branch on the adsorption branch, the distribution curve derived from the desorption data is preferred, and is described in this procedure. The computational method is essentially the procedure developed by Barrett, Joyner, and Halenda (**2**) except for the incorporation of a few simplifications.

NOTE 2—In cases where it has been established that the adsorption branch of the nitrogen isotherm is to be analyzed, the procedure proposed by Cranston and Inkley (**3**) is recommended.

5. Significance and Use

5.1 Pore volume distribution curves obtained from nitrogen sorption isotherms provide one of the best means of characterizing the pore structure in porous catalysts, provided that the limitations of the method are kept in mind. Used in conjunction with the BET treatment for surface area determination (**4**), these methods provide an indispensable means for studying the structure associated with pores usually important in catalysts. This practice is particularly useful in studying changes in a series of closely related samples caused by treatments, such as heat, compression, or extrusion often used in catalyst manufacturing. Pore volume distribution curves can often provide valuable information during mechanistic studies dealing with catalyst deactivation.

6. Computational Procedure

6.1 This procedure requires the use of a series of experimentally measured relative pressures $[P_4(i)/P_0(i)]$ and the corresponding quantities of nitrogen gas adsorbed $[V_{de}]$ expressed in units of cm³ STP/g. The experimental data required in the use of this procedure can be measured by following the steps outlined in Test Method D4222. Inspect the nitrogen sorption isotherm in the region above $P/P_o = 0.95$. If the solid contains no pores larger than 100 nm (1000 Å) radius, the isotherm remains nearly horizontal over a range of P/P_o approaching unity and it is a simple matter to select a starting relative pressure within this region, establishing an upper limit on the pore size range to be studied. If pores larger than 100 nm (1000 Å) are present however, the isotherm rises rapidly near $P/P_O = 1$ and the total pore volume cannot be well defined. This limiting adsorption can then be identified reliably only if the temperature is very carefully controlled and there are no "cold spots" in the apparatus (which lead to bulk condensation of the gas and a false measure of the adsorption in the volumetric method). Selecting the starting relative pressure for

the computational procedure is then made more difficult. In most cases a starting relative pressure of 0.99 will be suitable, which corresponds to an upper limit on pore size of 100 nm (1000 Å) in radius. If necessary, interpolate the values of V_{de} to determine the quantity of nitrogen gas adsorbed at the chosen starting relative pressure.

6.2 The procedure requires numerous arithmetical steps which can best be carried out with the aid of a work sheet. An example (**4**) of a form found useful in the calculations is provided in Table 1. List in descending order the experimentally determined relative pressures $[P_4(i)/P_0(i)]$ in Column 1, beginning with the value chosen as the starting relative pressure. Generally, values below a relative pressure of 0.25 will not be required in the calculations. Convert the uptake values into a liquid volume (mm³/g) by multiplying the value of V_{de} in cm³ STP/g with the conversion factor 1.5468 derived from $V_L = 34.67$ cm³/mole. List in Column 9 the corresponding quantities of nitrogen adsorbed.

6.3 For each relative pressure, calculate a value for the radius of the core, r_k, by means of the Kelvin equation,

$$RT \ln (P_4/P_0) = -\frac{2\gamma V_L}{r_k} \tag{1}$$

given in the form

$$r_k \ (\text{Å}) = -\frac{9.574}{\ln (P_4/P_0)} \tag{2}$$

with $T = 77.35$ K; $\gamma = 8.88$ mN/m; and $V_L = 34.67$ cm³/mole. List the values in Column 2. For each successive decrement in relative pressure, calculate $\bar{r}_k$, the mean of the values of r_k for the present and previous pressures, and list these mean values in Column 3.

6.4 The average thickness, t, of the multilayer film of nitrogen adsorbed on the walls of the pores at each relative pressure is used to calculate the amount of nitrogen desorbed from the film in pores which have lost their capillary condensate. For each relative pressure, calculate a value for the film thickness from the expression (**5**)

$$t \ (\text{Å}) = \left[\frac{13.99}{0.034 - \log (P_4/P_0)}\right]^{\frac{1}{2}} \tag{3}$$

and list the values in Column 4. For each successive decrement in relative pressure, calculate the differences in the values of t, and list these differences as Δt in Column 5.

6.5 Since a cylindrical pore model is assumed, the radius of the pore, r_p, is given by addition of the core radius, r_k, and the film thickness value, t. Add the values in Column 2 to the corresponding values in Column 4 and record the results in Column 6 as r_p. For each successive decrement in relative pressure, calculate $\bar{r}_p$, the mean of the pore radii, r_p, for the present and previous pressures, and record these values in Column 7.

6.6 Compute the quantity $(\bar{r}_p/\bar{r}_k)^2$ from the values listed in Columns 7 and 3. This quantity will be used later to correct the core volume to the volume for each group of pores. The core volume is the region within the pore that fills by capillary condensation of the nitrogen. List the computed values in Column 8 as the volume correction factor, Q.

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[3] The boldface numbers in parentheses refer to a list of references at the end of this standard.

TABLE 1 Pore Distribution Computational Work Sheet

Sample Identification _____   Date_____

1 P_4/P_O	2 r_k Å	3 $\bar{r}_k$ Å	4 t Å	5 Δt Å	6 r_p Å	7 $\bar{r}_p$ Å	8 Q	9 V_{de} mm³/g	10 ΔV_T mm³/g	11 ΔV_f mm³/g	12 ΔV_k mm³/g	13 ΔV_p mm³/g	14 ΔS_p m²/g	15 $\Sigma \Delta S_p$ m²/g	16 $\Sigma \Delta V_p$ mm³/g
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Column 2: $r_k = \dfrac{-9.574}{\ln(P_4/P_0)}$

Column 4: $t = \left[\dfrac{13.99}{0.034 - \log(P_4/P_0)}\right]^{1/2}$

Column 11: $\Delta V_f = 0.085 \cdot \Delta t \cdot \Sigma \Delta S_p$ (preceding line)

Column 12: $\Delta V_k = \Delta V_T - \Delta V_f$

Column 13: $\Delta V_p = \Delta V_k \times Q$

Column 14: $\Delta S_p = 20 \times (\Delta V_p / \bar{r}_p)$

Note 3—For a cylindrical pore $\bar{r}_p$ and $\bar{r}_k$ are related to Q by the exact expression:

$$Q = [\bar{r}_p/(\bar{r}_k + \Delta t)]^2 \qquad (4)$$

For $\bar{r}_k > 30$ Å, $\Delta t < 1\%$ $\bar{r}_k$. Simplifying Q by eliminating Δt gives $(\bar{r}_p/r_k)^2$.

6.7 The amounts of nitrogen desorbed for each successive decrement in relative pressure are calculated by progressive subtraction of the values of the amounts of nitrogen adsorbed, V_{de}, listed in Column 9 from the succeeding one. Compute these differences and list the values in Column 10 as, ΔV_T. Each value of ΔV_T except for the initial one in line 2 contains contributions from the amounts of nitrogen given up by loss of capillary condensate and by thinning of the nitrogen film adsorbed on the walls of pores which have previously released their capillary condensate. The initial value of ΔV_T is due solely to the amount of nitrogen contributed from loss of capillary condensate, since it is assumed that at the highest relative pressure all of the pores are completely filled with nitrogen, and that no thinning of the nitrogen film occurs for the first decrement in relative pressure.

6.8 In completing the calculation to obtain a value for the pore volume, ΔV_p, corresponding to each mean pore size, $\bar{r}_p$, of a group of pores, it will be necessary to work through the calculation for each line before proceeding to the next line. ΔV_f in Column 11 is the amount of nitrogen given up during desorption from thinning of the nitrogen film adsorbed on pore walls. For line 2, ΔV_f equals zero and the value of ΔV_T is assigned to ΔV_k, the volume of the inner core which fills by capillary condensation of the nitrogen. Multiply the value of ΔV_k by the corresponding volume correction factor, Q, listed in Column 8 to obtain ΔV_p. List the value in Column 13 of line 2.

6.9 Calculate the surface area of the pore walls contained in volume ΔV_p as follows:

$$\Delta S_p (\text{m}^2/\text{g}) = 20 \times \frac{\Delta V_p}{\bar{r}_p} \qquad (5)$$

Using the value of ΔV_p from Column 13 and the corresponding value of $\bar{r}_p$ from Column 7, compute a value for ΔS_p and list it in Column 14. A value for the total surface area of the pores that have become exposed is obtained by summation of the value for ΔS_p with other ΔS_p values in all preceding lines of Column 14. List the value of total area in Column 15 as $\Sigma \Delta S_p$. A value for the cumulative pore volume is obtained by summation of the value ΔV_p with other ΔV_p values in all preceding lines of Column 13. List the value of the cumulative pore volume in Column 16 as $\Sigma \Delta V_p$.

Note 4—The expression relating the surface area of the pore walls contained in volume ΔV_p is as follows:

$$\Delta S_p (\text{m}^2/\text{g}) = 2 \times \frac{\Delta V_p}{\bar{r}_p} \times 10^4 \qquad (6)$$

with ΔV_p in cubic centimetres and $\bar{r}_p$ in angstroms. Converting cubic

centimetres to cubic millimetres gives rise to the conversion factor of 20 in the preceding equation.

6.10 An essential feature of this test method is that the volume of nitrogen lost from the adsorbed multilayer film can be calculated from the total area of the pores exposed, $\Sigma\Delta S_p$, and the decrease in the thickness of the film, Δt. Using the value $\Sigma\Delta S_p$ from the preceding line, compute ΔV_f from the expression

$$\Delta V_f = 0.085 \times \Delta t \times \Sigma\Delta S_p \text{ (preceding line)} \qquad (7)$$

and record the value in Column 11. Subtract the value of ΔV_f from ΔV_T to obtain the value of ΔV_k of Column 12. Multiply the value of ΔV_k with the corresponding value of Q in Column 8 to obtain the value of ΔV_p of Column 13. Compute ΔS_p and $\Sigma\Delta S_p$ following the procedure in 6.9. Determine a value for $\Sigma\Delta V_p$.

NOTE 5— See Appendix X1 for a more detailed discussion on the calculation of values for ΔV_f and the source of the coefficient.

6.11 Repeat the calculations in 6.10 for each line down to a relative pressure lying between 0.25 and 0.30. If at an intermediate relative pressure, the value of ΔV_k should become equal to zero or negative, discontinue the calculation, and in all subsequent calculations use the data from the preceding line and above.

NOTE 6—It is generally agreed that the value of the pore size to which the Kelvin equation will no longer apply at the lower end of the pore size range varies from one adsorption system to another. As a result, difficulties arise in knowing precisely where to end the pore volume calculation for a particular nitrogen sorption isotherm. It is believed that the Kelvin equation will rarely be valid for a material below a pore size of 10 to 15 Å. We have chosen to terminate the pore volume calculation at a relative pressure lying between 0.25 and 0.30 which corresponds to pore size within the range from 11.6–13.0Å in radius. There are, however, other procedures that can be used to establish where the calculation should be terminated at the low relative pressure end of the isotherm. These alternative methods include:

(1) The use of the precise value for the relative pressure that corresponds to the closure point of the hysteresis loop at the lower end of the isotherm. This value may represent, in many cases, the point at which irreversible capillary condensation commences within the pore structure of the material.

(2) The use of values of $\Sigma\Delta V_p$, from Column 16 in Table 1 and the value of V_{de} from line 1 of Column 9. The calculation may be terminated at the point where the value of $\Sigma\Delta V_p$ becomes equal to or greater than the starting value of V_{de}. At this point in the calculation, the measured pore volume at the chosen starting relative pressure will have been exhausted. If the material is believed to be microporous, this method of termination shall not be used.

(3) A t-plot analysis (5) may be used to detect the beginning of capillary condensation. From a plot of the amounts of nitrogen adsorbed versus the t values determined from the equation in 6.4, the beginning of capillary condensation may be detected from the point of upward curvature in the plot. The relative pressure corresponding to this t value where the upward curvature commences is used to terminate the calculation. This method has the advantage of being able to detect the presence of reversible capillary condensation which is not accompanied by hysteresis and cannot be detected by visual examination of the isotherm. For precise work, this method of termination is highly recommended.

6.12 The last value of $\Sigma\Delta S_p$ computed in Column 15 is assigned to the total cumulative surface area. The last value of $\Sigma\Delta V_p$ computed in Column 16 is assigned to the total cumulative pore volume.

NOTE 7—A comparison of the value for the total cumulative surface area with that for the BET surface area determined from the low pressure region of the isotherm provides an empirical test of the internal consistency of the pore volume distribution calculations. If the assumed pore model is a true representation of the actual pore system, these values should show perfect agreement. Experience has shown that these two values often agree to within ±5 % but differences of ±20 % are not uncommon. If the BET surface area is substantially greater than the total cumulative surface area, the difference may be due to the presence of micropores. Results showing deviations greater than ± 20 % should be interpreted with caution.

7. Presentation of Data

7.1 The report shall consist of the following information:

7.1.1 Sample identification, and a statement indicating the use of the desorption isotherm and a cylindrical pore model.

7.1.2 A table or graph composed of the cumulative pore volume data obtained from the values in Column 16 and the corresponding pore sizes, r_p, given in Column 6. A graphical, cumulative pore volume plot can be constructed by plotting the $\Sigma\Delta V_p$ values in cubic millimetres per gram on the ordinate with an arithmetic scale and the corresponding pore size r_p in Å on the abscissa with a logarithmic or arithmetic scale, as appropriate. This cumulative distribution will generally appear as a smooth line and should reveal points of inflection where the pore volume is changing most rapidly. An example of a cumulative plot is shown in Fig. X1.1.

7.1.3 The report may also include a differential plot of the distribution. This can be constructed from slopes derived from the smoothed cumulative plot at appropriate pore radius intervals ($\Delta r_p{}'s$), or from values of $\Delta V_p/\Delta r_p$ calculated from the data in Columns 6 and 13 of Table 1 and the corresponding values of the mean pore size, $\bar{r}_p$, from Column 7. The change in slope shall be plotted on the ordinate and the corresponding mean pore size on the abscissa. Arithmetic or logarithmic scales may be used on the ordinate and abscissa, as appropriate. An example of a differential plot is shown in Fig. X1.2.

8. Precision and Bias

8.1 This practice determines in suitable cases how the pore volume is distributed as a function of pore size for mesoporous solids. These calculated pore volumes depend on the branch of the nitrogen isotherm used and the pore model assumed. They also vary depending on the precise values used for the film thickness and on how the film thickness corrections are determined. Thus, no statement as to the precision of this practice is possible.

8.2 *Bias*—No estimate of the bias of this practice is possible.

9. Keywords

9.1 catalysts; nitrogen adsorption; pore size distribution

APPENDIX

(Nonmandatory Information)

X1. CALCULATION OF CORRECTIONS FOR THE VOLUME RELEASED DURING THINNING OF THE PHYSICALLY ADSORBED LAYERS

X1.1 According to Barrett, Joyner, and Halenda (2), corrections for the volume of nitrogen lost during the thinning of the adsorbed multilayer film, $\Delta V_f(i)$, are given by the expression:

$$\Delta V_f(i) = \Delta t(i) \sum_{j=1}^{i-1} C(j) \Delta S_p(j) \qquad (X1.1)$$

where:

j	=	sequence number of desorption steps,
t	=	adsorbed layer thickness,
$C(j)$	=	$\dfrac{(\bar{r}_p(j) - \bar{t}(j))}{\bar{r}_p(j)}$
$\bar{r}_p(j)$	=	average pore radius of the pore group emptied in the jth desorption step,
$\bar{t}(j)$	=	average adsorbed layer thickness in the jth desorption step, and
$\Delta S_p(j)$	=	surface area of the pore group emptied in the jth desorption step.

X1.2 See Fig. X1.1 and Fig. X1.2. The value of C varies for the different pore groups, and since t is a function of relative pressure, the value of C for a particular pore group varies stepwise with relative pressure. These variations in the C values make the calculation of ΔV_f from the above expression very time consuming. In practice, it has been shown (2) that a very close approximation to the correct pore size distribution can be obtained by using a constant C value for a particular adsorbent. Since the C value is known to vary between the limits (2) of 0.72 to 0.97 for pores of any size, we have chosen a value of 0.85 which is midway between the limits of C in deriving the equation for ΔV_f in 6.10. Other constant values of C may be used in place of 0.85 by substituting the selected C value into the equation

$$\Delta V_f = C \times 0.1 \times \Delta t \times \Delta S_p \text{ (preceding line)} \qquad (X1.2)$$

and replacing the equation in 6.10 with the above. For precise work, it is recommended that the value of C be allowed to vary. This will require the redetermination of the C value for each pore group at the various relative pressures used in the computation.

NOTE X1.1—Because of the time and effort involved in the use of variable C values, it is desirable to facilitate the computations with the aid of a computer.
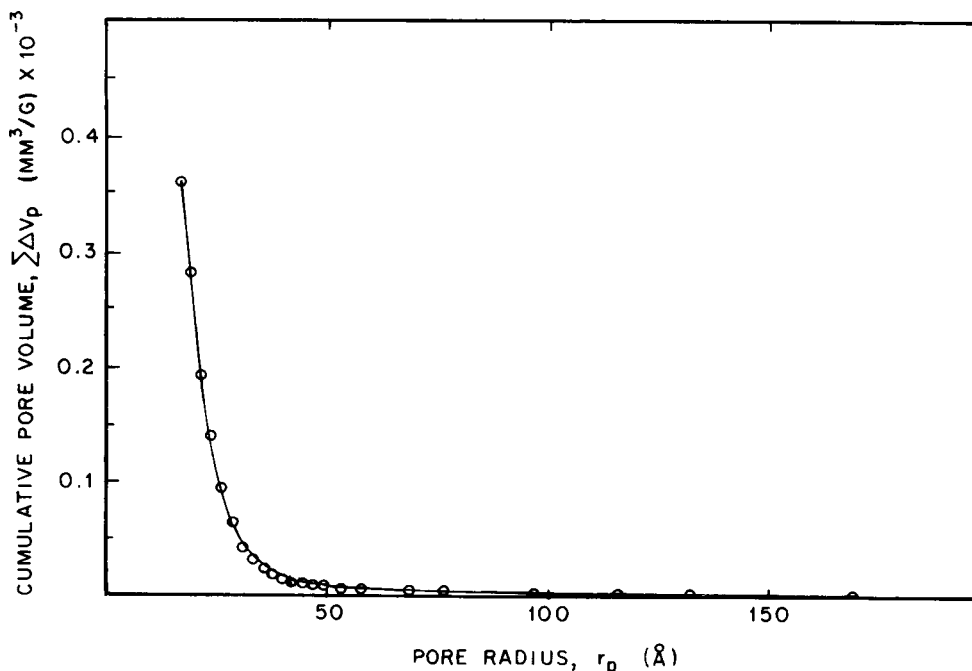

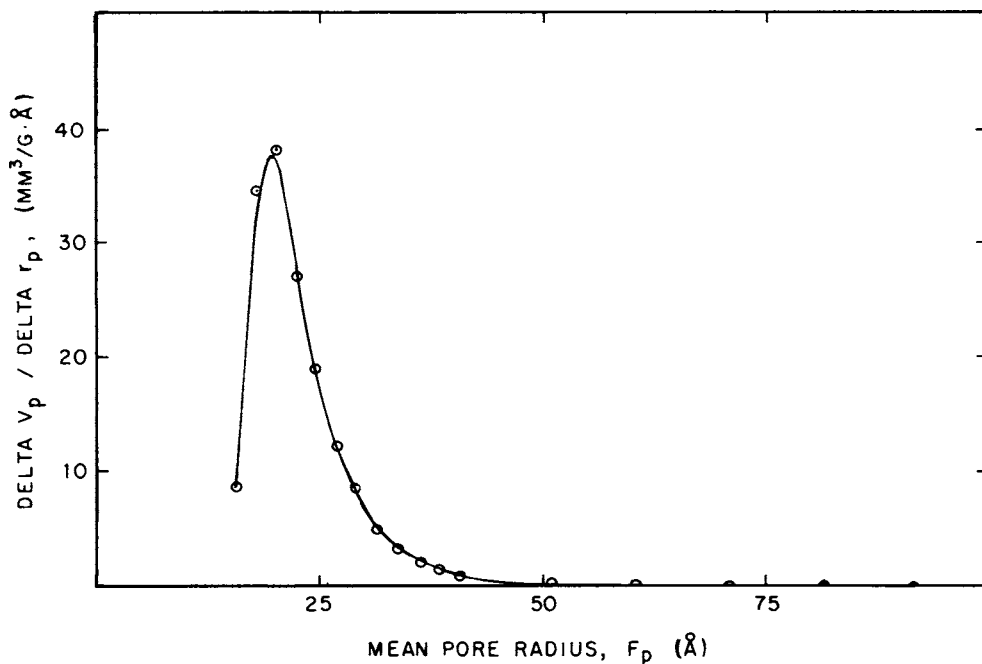FIG. X1.1 Example of Cumulative Pore Volume Plot on a Silica-Alumina Cracking Catalyst

FIG. X1.2 Example of a Differential Plot on a Silica-Alumina Cracking Catalyst

REFERENCES

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